May 6th, 2026
First Dutch Workshop for
Combatting Crime,
Amsterdam,

A Hidden Formula for Fraud? Equation Learning for Fraud Detection

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Informatics Institute
Institute for Logic, Language and Computation

Today's Roadmap

- Introduction
- What is Symbolic Regression?
- Two Research Papers
  - Explainable Fraud Detection through Deep Symbolic Classification (XAI 2024)
  - ECSEL: Explainable Classification via Signomial Equation Learning (ICML 2026)
- Final Takeaways

A brief introduction

"Who Am I? Ah, that's the greatest puzzle" — Lewis Carroll

Erman Acar (Assistant Professor for Safe & Explainable AI in Finance)

Affiliation: Informatics Institute & Institute of Logic, Language and Computation, at UvA

Research: XAI, Neurosymbolic Systems, Multi-agent Systems, Causality

Role: Leading Finesse Lab

FINESSE LAB

Finance-inspired Neurosymbolic Systems for Safety & Explainability

?

Adia Lumadjeng
PhD Student

Andreas Sauter (VU)
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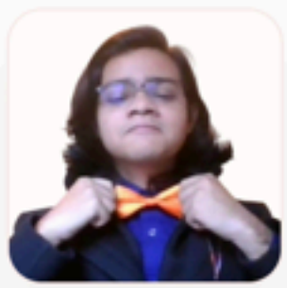


Satchit Chatterji
PhD Student

Finesse Lab

Finance-inspired Neurosymbolic Systems
for Safety & Explainability

Qualities

Research Theme

Safety & Control | Circuit-Level Mechanisms | Explainability by design

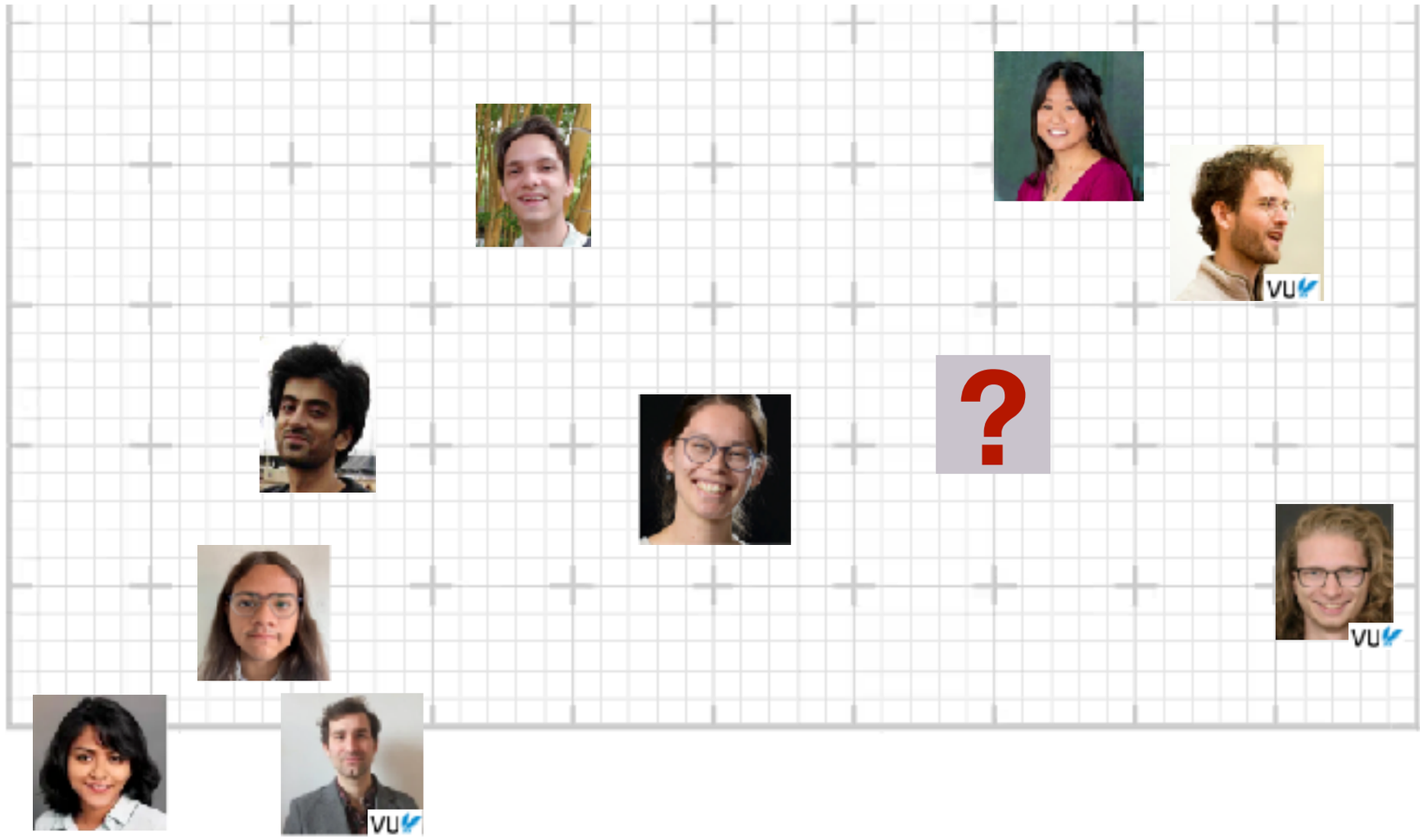
Equation Discovery

Causality-driven AI/
Fraud Detection

Foundation Models
Forecasting

Multimodal Reasoning

Multiagent Systems



Finesse Lab

Finance-inspired Neurosymbolic Systems
for Safety & Explainability

Promoting socially responsible, financially compliant AI Research.

For People

Starting from societal and stakeholder needs, we try to address fairness, transparency, accountability, and recourse.

By People

We integrate human expert knowledge directly into AI through rules, logic, policy constraints, and stakeholder values.

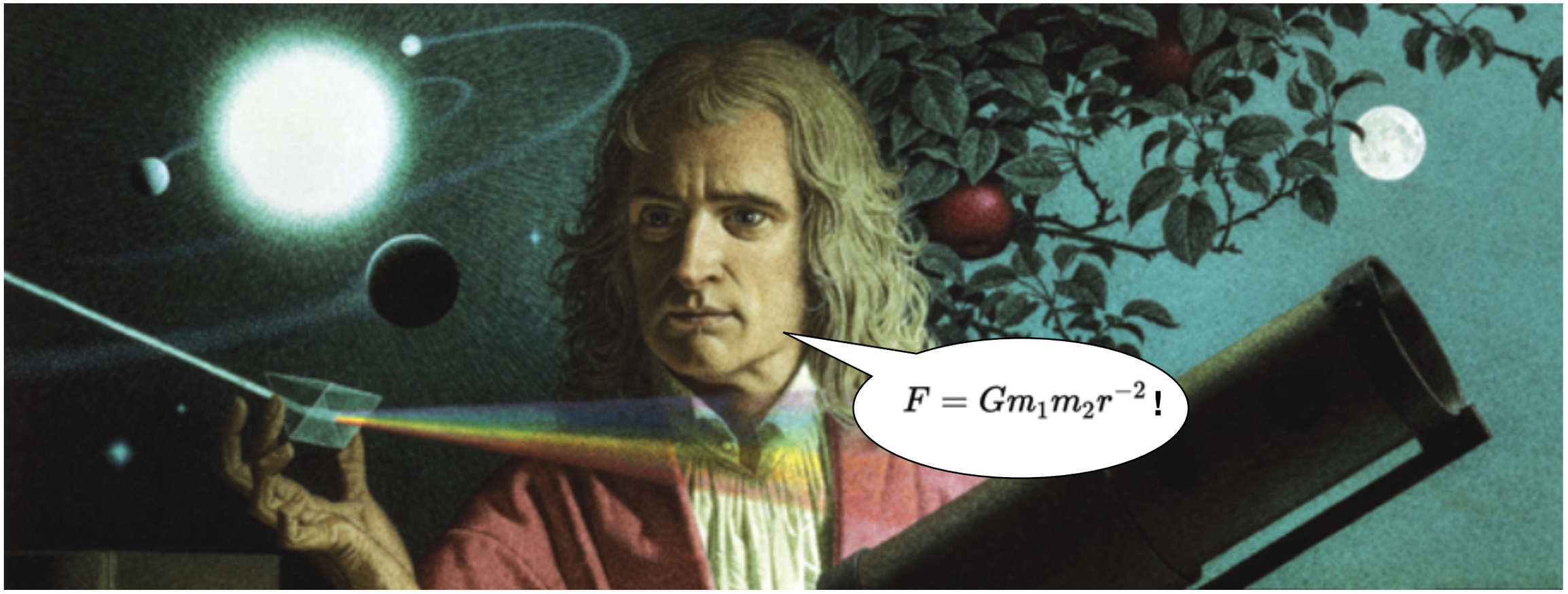
With People

Keeping humans in the loop, we see AI as a support for analysts, auditors, and citizens, not a black-box replacement.

Finesse is built on SIAS principles.

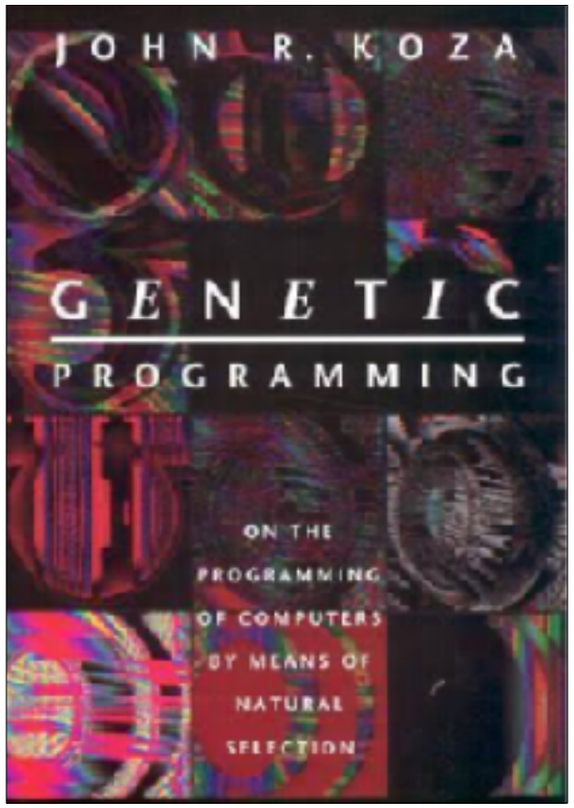
What is Symbolic Regression?

What is the law behind the data?

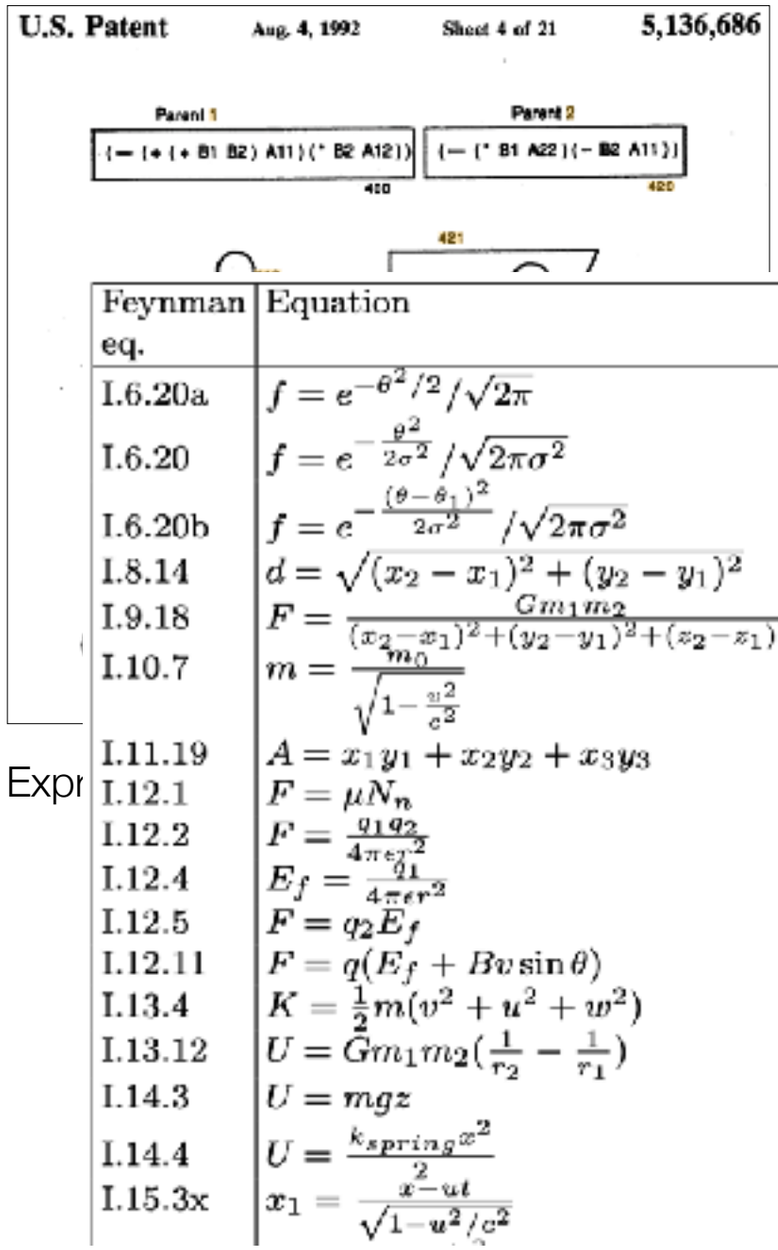


G	m_1 kg	m_2 kg	r m	F N
6.674×10^{-11}	10	20	2	3.337×10^{-9}
6.674×10^{-11}	10	20	4	8.343×10^{-10}
1.000×10^{-10}	30	20	2	1.500×10^{-8}
5.000×10^{-11}	30	40	3	6.667×10^{-9}
8.000×10^{-11}	50	40	5	6.400×10^{-9}
4.000×10^{-11}	100	80	10	3.200×10^{-9}

Symbolic Regression:
An old forgotten child of Machine Learning



Richard Feynman (1918-1988)



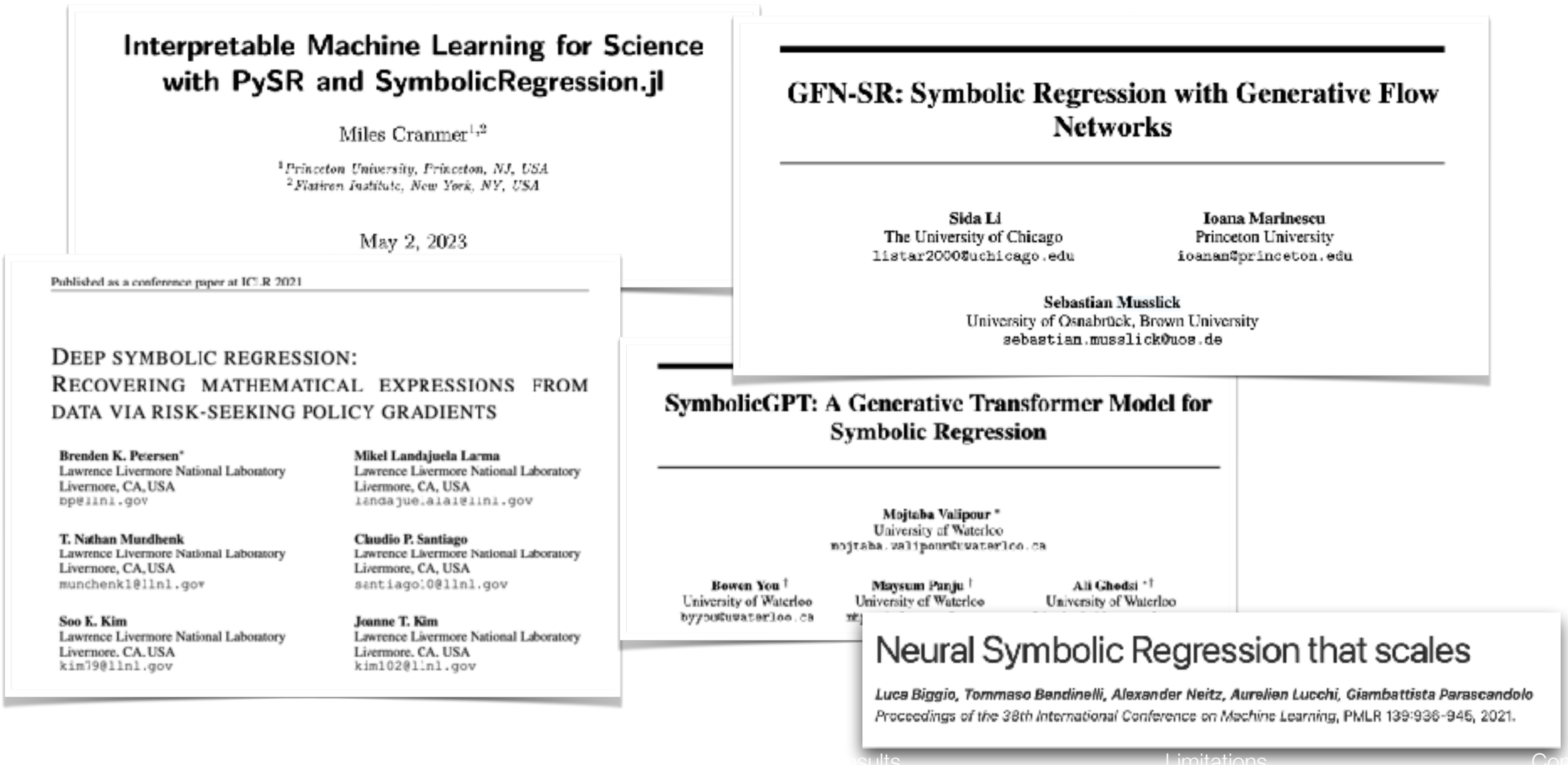
Feynman eq.	Equation	Solution time (s)
I.6.20a	$f = e^{-\theta^2/2}/\sqrt{2\pi}$	16
I.6.20	$f = e^{-\frac{\theta^2}{2\sigma^2}}/\sqrt{2\pi\sigma^2}$	2992
I.6.20b	$f = e^{-\frac{(\theta-\theta_1)^2}{2\sigma^2}}/\sqrt{2\pi\sigma^2}$	4792
I.8.14	$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$	544
I.9.18	$F = \frac{Gm_1m_2}{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$	5975
I.10.7	$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$	14
I.11.19	$A = x_1y_1 + x_2y_2 + x_3y_3$	184
I.12.1	$F = \mu N_n$	12
I.12.2	$F = \frac{q_1q_2}{4\pi\epsilon r^2}$	17
I.12.4	$E_f = \frac{q_1}{4\pi\epsilon r^2}$	12
I.12.5	$F = q_2E_f$	8
I.12.11	$F = q(E_f + Bv\sin\theta)$	19
I.13.4	$K = \frac{1}{2}m(v^2+u^2+w^2)$	22
I.13.12	$U = Gm_1m_2(\frac{1}{r_2}-\frac{1}{r_1})$	20
I.14.3	$U = mgz$	12
I.14.4	$U = \frac{k_{spring}x^2}{2}$	9
I.15.3x	$x_1 = \frac{x-ut}{\sqrt{1-u^2/c^2}}$	22

x_1	x_2	x_3	y
0.00	1.00	2.00	6.00
1.57	0.50	4.00	7.00
3.14	2.00	1.00	6.00
0.79	3.00	0.50	5.21

$$y = \sin(x_1) + 3x_2x_3$$

AI Feynman Dataset:
An arena for Symbolic Regressors

Blooming of Symbolic Regression



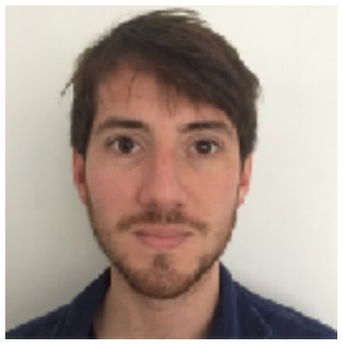
This is a difficult problem: *Symbolic Regression is NP-hard, M. Virgolin, S. Pissis, 2022*

First Research Paper:

Explainable Fraud Detection via Deep Symbolic Classification

(XAI 2024)

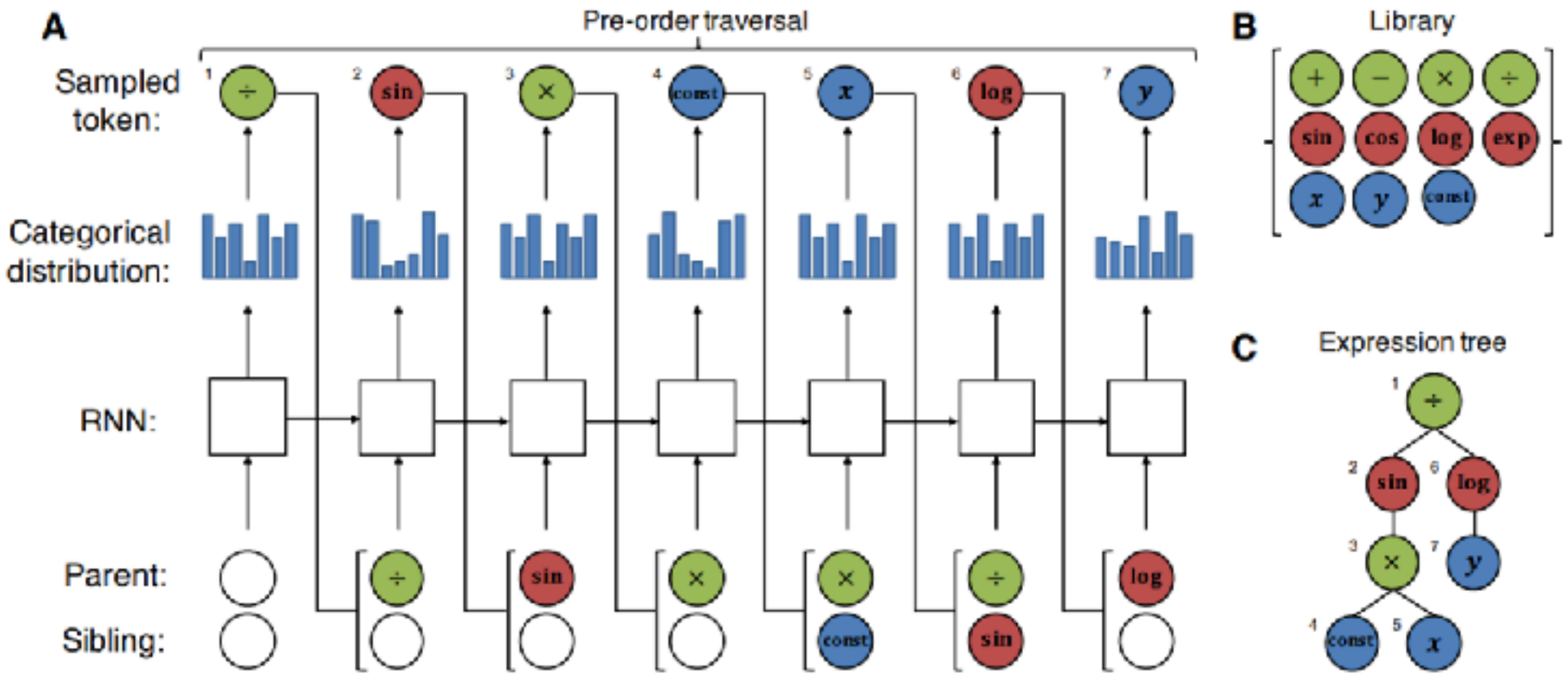
Samantha Visbeek
(Zanders)



Floris den Hengst (ING & VU)

E. A.

Deep Symbolic Regression



- Elements are sampled from a categorical distribution emitted by the RNN (**A**), on the library L of elements in (**B**).
- The parent and sibling nodes of the next element are the next input to the RNN.
- The sampling process ends when all branches reach the leaf nodes.
- The resulting list [÷, sin, ×, *constant*, x, log, y] is the preorder traversal of the syntax tree that represents $\sin(cx)/\log(y)$ (**C**).
- The syntax tree (**C**) that can be reconstructed from the preorder traversal list from (**A**).

Image taken from Petersen et al. 2021

Dataset
(PaySim)

Synthetic Financial Datasets For Fraud Detection

Synthetic datasets generated by the PaySim mobile money simulator

Data Card | Code (258) | Discussion (29) | Suggestions (0)

About Dataset

Context

There is a lack of public available datasets on financial services and specially in the emerging mobile money transactions domain. Financial datasets are important to many researchers and in particular to us performing research in the domain of fraud detection. Part of the problem is the intrinsically private nature of financial transactions, that leads to no publicly available datasets.

We present a synthetic dataset generated using the simulator called PaySim as an approach to such a problem. PaySim uses aggregated data from the private dataset to generate a synthetic dataset that resembles the normal operation of transactions and injects malicious behaviour to later evaluate the performance of fraud detection methods.

Content

PaySim simulates mobile money transactions based on a sample of real transactions extracted from one month of financial logs from a mobile money service implemented in an African country. The original logs were provided by a multinational company, who is the provider of the mobile financial service which is currently running in more than 14 countries all around the world.

Usability ⓘ
9.82

License
CC BY-SA 4.0

Expected update frequency
Not specified

Tags
Finance | Crime

- PaySim is a data set of simulated transactions based on proprietary real transactions
- developed to provide researchers that exhibits statistical properties similar to a real payment transaction data set
- Contains ~ 6.3 million transactions over a period of a month
- Highly Imbalanced: with a fraudulent transaction rate of ~ 0.13%.

https://www.kaggle.com/datasets/ealaxi/paysim1/data

Results

method	accuracy	precision	recall	F1 score
RF + RUS	0.93	0.02	0.93	0.03
XGBoost + RUS	0.95	0.02	**0.94**	0.05
k-NN + RUS	0.94	0.02	0.83	0.03
SVM + RUS	0.95	0.02	0.70	0.03
RF	**0.99**	**0.99**	0.67	0.81
XGBoost	**0.99**	0.98	0.70	**0.82**
DSC (average)	**0.99**	0.95 (.01)	0.67	0.78
DSC (best expression)	**0.99**	0.95	0.67	0.78

- Test set averaged over 5 runs: the baseline classification models with and without Random Undersampling (RUS)
- Split: 75% for training, 15% for validation, 10% for test.
- The reward function r_{F1} and a threshold value of 0.8 and the best expression obtained by DSC
- Undersampling (RUS) yields a loss in excessive amount of information and causes overfitting.
- k-NN and SVM trained only with RUS. Otherwise, timeout after 50 hours due to high-class imbalance.

Pareto Front of Prediction vs. Complexity (Occam's Razor)

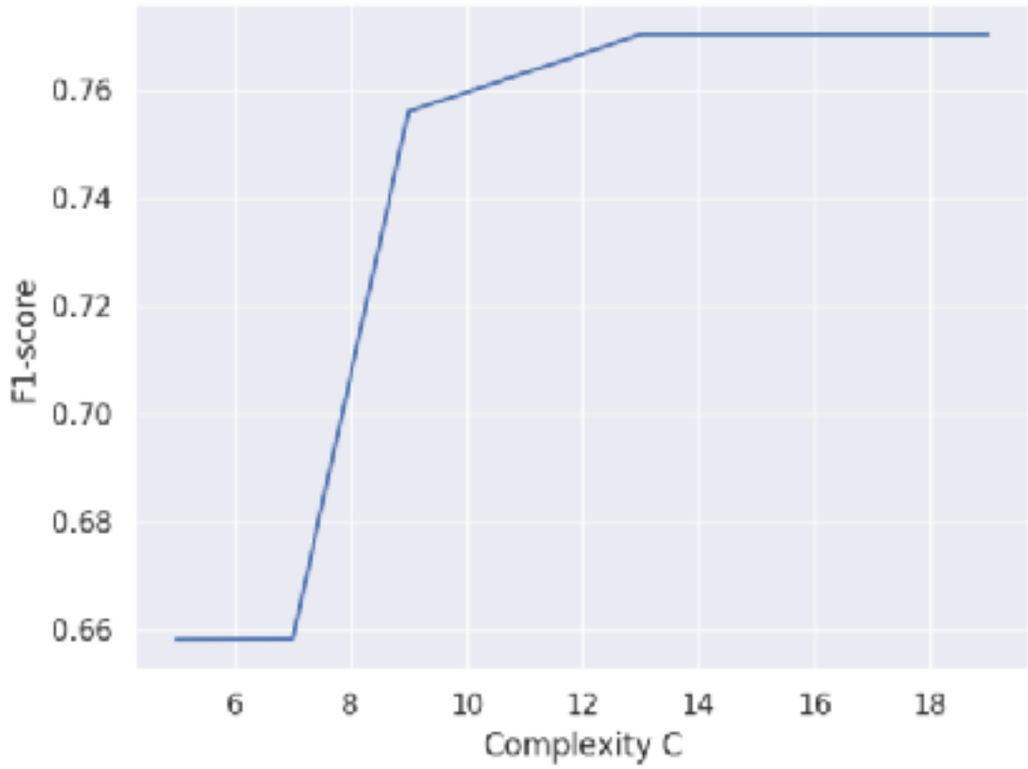
Complexity of Tokens

token τ	complexity c
+, -, ×, feature, constant	1
÷, square	2
sin, cos	3
exp, log, square-root	4

Complexity of an Expression

$$C(f) = \sum_{i=0}^{T} c(\tau_i)$$

Pareto Front of Performance vs. Complexity



$$t = 0.\ r_{F1} = \frac{2pr}{p+r}$$

The optimal expression selected w.r.t. adding more complexity to the expression does not result in a sufficient increase in the F1 score.

- prevents overfitting
- ensures that the expressions are not overly complex

Understanding the Best Expression

$$f = \sqrt{\text{externalDest} + \text{type\_cash-out}} \cdot (\text{amount} - \text{maxDest7} + \text{type\_transfer})$$

A B

Complexity: 13

Boolean features: externalDest, type_cash-out, type_transfer

Non-negative numerical Features: amount, maxDest7

Complexity of Tokens

token τ	complexity c
+, -, ×, feature, constant	1
÷, square	2
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exp, log, square-root	4

Complexity of an Expression

$$C(f) = \sum_{i=0}^{T} c(\tau_i)$$

Decision Rule: $\hat{y} = 1(\text{fraud}), \text{if } \sigma(f) > 0.7,$ (since $t = 0.7$)

$\hat{y} = 1(\text{fraud}), \text{if } f > 0.85.$ rewriting w.r.t. $\sigma(f) = 1/(1 + e^{-f})$

Note that, (1) amount – maxDest7 ≤ 0, since maxDest7 includes current transaction

(2) type_cash-out and type_transfer are mutually exclusive (hence one-hot encoding)

Two Scenarios: **type_cash-out = 1:**

Then, type_transfer = 0, hence **B** ≤ 0. But since, externalDest ∈ {0,1}, $f \leq 0$, means no fraud.

type transfer_ = 1

Then, type_cash-out = 0, if externalDest = 0, **A** = 0, and no fraud.

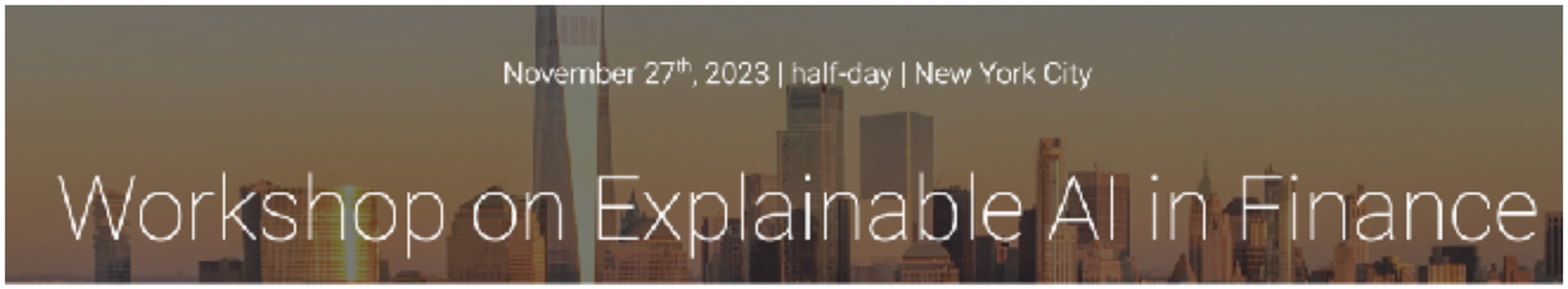
if externalDest = 1, fraud = 1 *iff* $f > 0.85$

iff **B** > 0.85

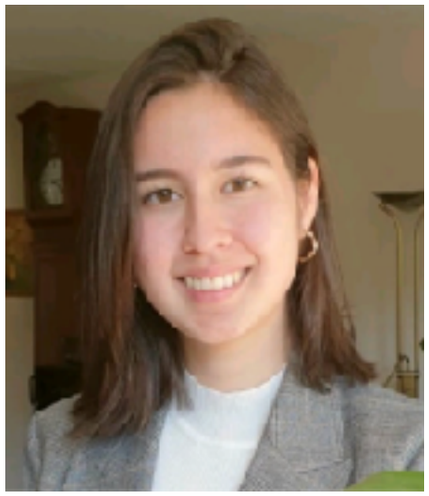
iff amount - maxDest7 > -0.15

Derived Decision Rule: *fraud* if type = *transfer* ∧ externalDest = *true* ∧ amount - maxDest7 > -0.15, *legitimate* otherwise.

It was a Master Thesis



(Accepted and presented)



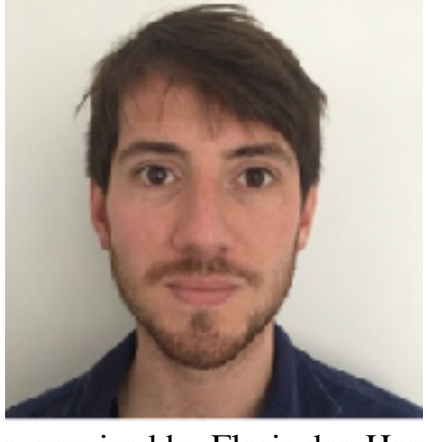
Samantha Visbeek

amsterdam ai

Call for Nominations is Open – Amsterdam AI Thesis Awards

The Amsterdam AI Thesis Awards aim to promote excellence in AI and Data Science from students at the Bachelor and Master level in Amsterdam-based Amsterdam-AI university partners (HvA, UvA, and VU).

The thesis won the Amsterdam AI Thesis Awards 2023



Co-supervised by Floris den Hengst (ING)

The 2nd World Conference on eXplainable Artificial Intelligence

17-19 July, 2024, Malta, Valletta

Published in XAI World conference 2024

Second Research Paper:

ECSEL: Explainable Classification via Signomial Equation Learning (ICML 2026 - to appear)

Adia Lumadjeng

PhD on Explainable AI for Fraud Detection

Business Analytics (ABS)

Socially Intelligent Artificial Systems Group (lvl)

Ilker Birbil.

Professor of AI & Optimisation Techniques for Business & Society

Business Analytics (ABS)

E.A.

A Closer Look at AI Feynman data set

Richard Feynman (1918-1988)

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I.14.4	$U = \frac{k_{spring}x^2}{2}$	9
I.15.3x	$x_1 = \frac{x-ut}{\sqrt{1-u^2/c^2}}$	22

I.12.2: $F = \frac{1}{4\pi} \cdot q_1 \cdot q_2 \cdot \epsilon^{-1} \cdot r^{-2}$

I.13.4: $K = \frac{1}{2}mv^2 + \frac{1}{2}mu^2 + \frac{1}{2}mw^2$

$$z(x) = \sum_{k=1}^{K} \alpha_k \prod_{j=1}^{m} x_j^{\beta_{k,j}}$$

signomial function

Udrescu, S.-M. And Tegmark, M. (2020): 'AI Feynman: A physics-inspired method for symbolic regression'. *Science Advances*, 6:eaay2631

45% are signomial functions!

Not only Physics: Signomials are everywhere!

Field	Famous model	Signomial form
Economics	Cobb–Douglas production / utility	$Y = AK^{\alpha}L^{\beta}$. This is a single monomial, so it is also a signomial. Used for output as a function of capital and labor; generalized versions use $\prod_i x_i^{\lambda_i}$. Wikipedia
Economics / optimization	Generalized cost or production functions	Sums such as $c_1K^aL^b + c_2E^d + c_3M^e$. These are classic signomial-programming objects when coefficients/exponents are unrestricted. PMC +1
Biology	Kleiber's law / allometric scaling	$B = aM^{3/4}$. A monomial relating metabolic rate to body mass; the exponent is debated, but the power-law form is canonical. PMC +1

Expressivity Result:

Theorem 3.1 (Universal Approximation for Signomials). *Let $D \subset \mathbb{R}^n_{>0}$ be a compact subset of the positive orthant. Then the set of signomials*

$$\mathcal{S} = \left\{ S : S(x_1, \dots, x_n) = \sum_{k=1}^{K} \alpha_k \prod_{j=1}^{n} x_j^{\beta_{kj}}, \quad K \in \mathbb{N},\ \alpha_k \in \mathbb{R},\ \beta_{kj} \in \mathbb{R} \right\} \tag{9}$$

is dense in $C(D, \mathbb{R})$. That is, for any continuous function $f : D \to \mathbb{R}$ and any $\epsilon > 0$, there exists a signomial $S \in \mathcal{S}$ such that

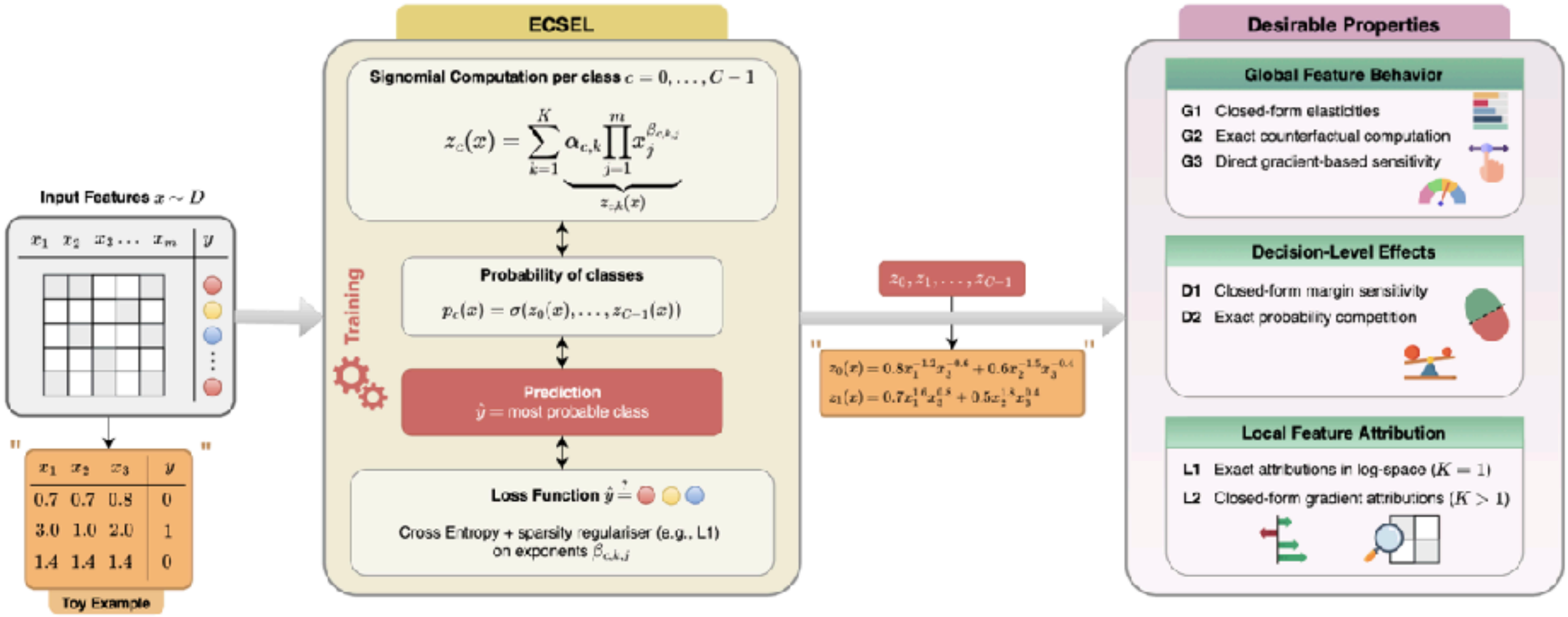
$$\sup_{(x_1,\dots,x_n)\in D} |f(x_1, \dots, x_n) - S(x_1, \dots, x_n)| < \epsilon. \tag{10}$$

[illegible]	[illegible] response	[illegible] Wikipedia
Physics	Newtonian gravity / inverse-square laws	$F = Gm_1m_2r^{-2}$. A monomial signomial with a negative exponent.
Physics / chemistry	Power-law rate laws	$r = kA^{\alpha}B^{\beta}$. Monomial signomial; common in reaction kinetics and transport modeling.
Engineering	Drag or scaling laws	$F_D = cv^2$, $P = cv^3$, or multi-term approximations such as $av^2 + bv + c$. Polynomial-like models are signomials because integer powers are allowed.

For a good reason: they are easy to compact, easy to interpret and expressive!

ECSEL

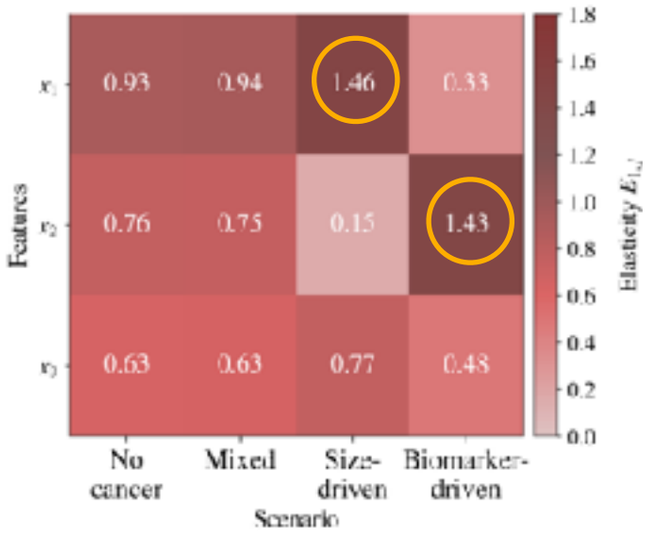
(Explainable classification via Signomial Equation Learning)



Online Shopping Intention Example

$$z = 0.10 \cdot \frac{PageValues^{0.47} \cdot Month^{0.07} \cdot PageValue\_per\_ExitRate^{1.09} \cdot ShopIntensity^{0.66}}{ExitRates^{0.41} \cdot Administrative^{0.14} \cdot IsReturn^{0.04}}$$

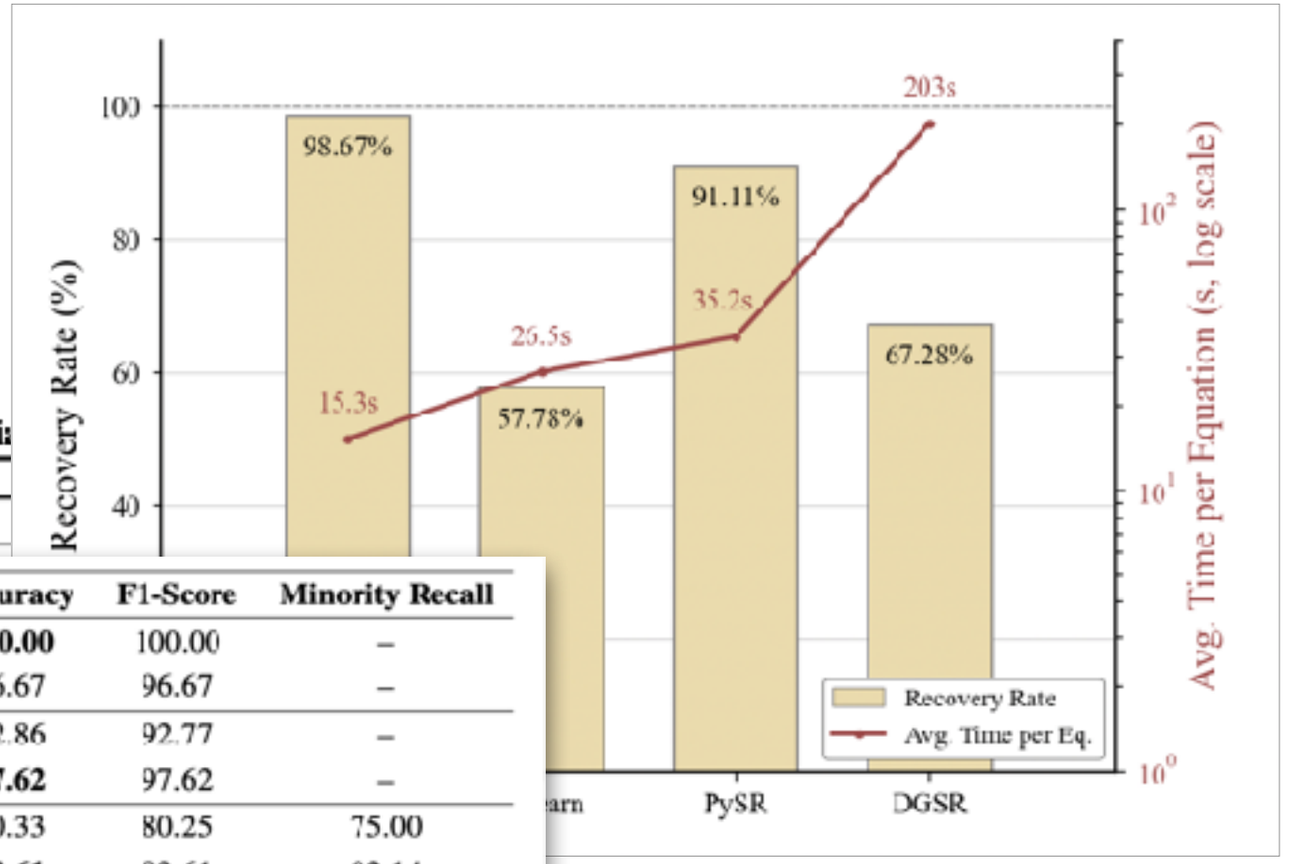
Interpretability Properties: Example



(a) **Feature importance**

Symbolic Regression

ECSEL vs. Generative Models



ECSEL: Explainable Classification via Signomi

Eq.	Expression
I.25.13	$V_e - \frac{q}{C}$
I.29.4	$k = \frac{\omega}{c}$
I.32.5	$P = \frac{q^2 a^2}{6\pi\varepsilon c^3}$
I.34.8	$\omega = \frac{qvB}{p}$
I.34.27	$E = \frac{p}{h\omega}$
I.38.12	$r = \frac{4\pi\varepsilon\hbar^2}{mq^2}$
I.39.10	$E = \frac{3}{2}pPV$
I.39.22	$P_P = \frac{nk_bT}{V}$
I.43.16	$v = \frac{\mu_{drift}qV_e}{d}$
I.43.31	$D = \omega_c k_b T$
I.47.23	$c = \sqrt{\frac{\gamma pr}{\rho}}$
II.3.24	$F_E = \frac{P}{4\pi r^2}$
II.4.23	$V_e = \frac{q}{4\pi\varepsilon r}$
II.8.7	$E = \frac{3}{5}\frac{q^2}{4\pi\varepsilon r}$
II.8.31	$E_{den} = \varepsilon E_f^2$
II.11.20	$P_e = \frac{n_p p_d^2 E_f}{3k_b T}$
II.13.17	$B = \frac{1}{4\pi\varepsilon c^2}\frac{2I}{r}$
II.27.16	$F_E = \varepsilon c E_f^2$
II.27.18	$E_{den} = \varepsilon E_f^2$
II.34.2A	$I = \frac{qv}{2\pi r}$
II.34.2	$\mu_M = \frac{qvr}{2m}$
II.34.11	$\omega = \frac{gqB}{2m}$

Dataset	Method	Accuracy	F1-Score	Minority Recall
IRIS	SVM	**100.00**	100.00	–
	ECSEL	96.67	96.67	–
SEEDS	LR	92.86	92.77	–
	ECSEL	**97.62**	97.62	–
HEARTS	LR	80.33	80.25	75.00
	ECSEL	**83.61**	83.61	82.14
ILPD	XGBoost	72.41	63.03	6.06
	ECSEL	**75.86**	74.39	42.42
TRANSFUSION	XGBoost	**80.06**	78.72	38.89
	ECSEL	79.33	77.95	41.67
CONTRACEPTIVE	XGBoost	**60.00**	59.14	–
	ECSEL	56.27	55.94	–
COMPAS	XGBoost	68.18	68.08	62.54
	ECSEL	**68.47**	68.36	62.82
DEFAULT	SVM	**81.82**	79.35	33.61
	ECSEL	81.74	79.34	34.06
SKINNONSKIN	XGBoost	**99.96**	99.96	99.97
	ECSEL	99.25	99.25	99.88
MAMMOGRAPHY	XGBoost	**98.70**	98.61	59.62
	ECSEL	98.66	98.57	59.62
LOAN	XGBoost	**99.51**	99.50	95.55
	ECSEL	99.22	99.21	92.97

100	0.55 ± 0.48	100	145.80 ± 32.03	100	25.51 ± 33.91	–	–
100	0.52 ± 0.45	100	121.01 ± 97.13	100	4.04 ± 16.88	–	–
						0	250.22 ± 2.05
						–	–
						–	–
						100	25.99 ± 1.29
						0	260.26 ± 14.51
						0	255.24 ± 1.02
						0	262.06 ± 4.22
						0	253.97 ± 5.09
						100	23.10 ± 0.14
						–	–
						–	–
						100	23.24 ± 0.81
						100	0.60 ± 0.02
						0	274.73 ± 6.11
						–	–
						–	–

Back to the Fraud Detection (PaySim)

Output

Performance

Methodl	F1 (%)	Recall (%)	Precision (%)	ROC-AUC	Threshold	Time (s)
Logistic Regression	55.44	51.06	60.66	0.9593	1.000	1642.8
Random Forest	88.50	84.01	93.50	0.9992	0.417	204.0
XGBoost	**89.90**	**87.82**	92.09	**0.9998**	0.989	**8.5**
ECSEL	79.08	68.10	**94.27**	0.9914	0.904	960.0

Equation ($p > 0.904$)

$$z = -0.07 \cdot \frac{A^{0.02} \cdot P^{0.03}}{exO^{0.03} \cdot exD^{0.16} \cdot CO^{0.14} \cdot \mathrm{T}^{0.06}\, D^{0.03}} + 0.09 \cdot \frac{OBO^{1.42}}{NBO^{0.04} \cdot exD^{0.07} \cdot CO^{0.06} \cdot D^{0.06}\, P^{0.05}}.$$

Feature name	Feature description	Type	Scaled	Notes
Original numerical features				
step	Discrete time step of the transaction (1 hour per step)	Continuous	Yes	Transaction timestamp
amount	Transaction amount	Continuous	Yes	Log-transformed
oldbalanceOrg	Origin account balance before transaction	Continuous	Yes	Log-transformed
newbalanceOrig	Origin account balance after transaction	Continuous	Yes	Log-transformed
oldbalanceDest	Destination account balance before transaction	Continuous	Yes	Log-transformed
newbalanceDest	Destination account balance after transaction	Continuous	Yes	Log-transformed
Transaction type indicators (one-hot encoded)				
CashIn (CI)	Cash-in transaction indicator	Binary	No	One-hot encoded
CashOut (CO)	Cash-out transaction indicator	Binary	No	One-hot encoded
Debit (D)	Debit transaction indicator	Binary	No	One-hot encoded
Payment (P)	Payment transaction indicator	Binary	No	One-hot encoded
Transfer (T)	Transfer transaction indicator	Binary	No	One-hot encoded
Engineered features				
pct balance taken (PctBT)	Fraction of the origin balance transferred in the transaction	Continuous	Yes	Clipped to $[0, 1]$
externalOrig	Indicator that the origin account is external (both balances equal zero)	Binary	No	Transaction-level rule

Final Takeaways

- Symbolic Regressions opens domain-specific explainable AI expertise to explain data.
- Not only interesting for science, but also public sector, and many businesses both in regression and classification.
- So instead of a black box, you get a human-readable formula that explains exactly what the model is doing, by construction.
- Once the equation is obtained, easy to verify, on par with industry choice methods in accuracy.
- It is objective; so no human bias, but you can impose still the domain knowledge (part of future research).
- No post-hoc approximations, no proxy explanations, , no under or over sampling, the model *is* the explanation.
- The desirable properties imply more consistent and faster computation compared to post-hoc methods.
- The secret sauce: signomials, expressive enough to capture complex patterns, simple enough to read.
- More application on real-world data

Thanks!

Thank you! Link to paper